Advanced Microeconomics

Midterm Winter 2011/2012

28th November 2011

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

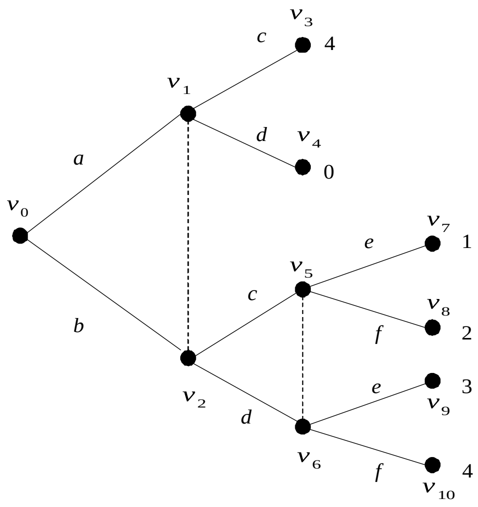
UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	8	$\sum$

Problem 1 (14 points)
Consider the following decision problem without moves by nature!



a) How many subtrees does this decision tree have?

b) Show that this decision situation exhibits imperfect recall!

c) How many strategies can you find? Give two examples.

d) Find the optimal strategies!

Solution:

a) There is one subtree, the whole tree.

b) $I(v_1) = I(v_2)$, but $X(v_1) = \{v_0, a, \{v_1, v_2\}\} \neq \{v_0, b, \{v_1, v_2\}\} = X(v_2)$ which implies imperfect recall.

c) Every strategy has an action at each information set. There exist 3 information sets. Therefore, every strategy is a tupel of 3 actions. Since there are two actions at each information set, we have 2^3 strategies, for example $[a, c, e]$ and $[a, c, f]$.

d) The optimal strategies are those that lead to a utility of 4. Therefore we look for those strategies that provoke the nodes v_3 or v_{10}. These are the strategies $[a, c, e]$, $[a, c, f]$ and $[b, d, f]$.

Problem 2 (6 points)
Consider a decision problem in strategic form with three strategies s_1, s_2, and s_3. Consider two mixed strategies $\sigma_1 = \left(\frac{1}{4}, \frac{3}{4}, 0\right)$ and $\sigma_2 = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. For a given state of the world $w \in W$, assume $\sigma_1 \in \sigma^{R,W}(w)$ and $\sigma_2 \notin \sigma^{R,W}(w)$. Find $s^{R,W}(w)$ and $\sigma^{R,W}(w)$!

Solution:

Optimal mixed strategies mix optimal pure strategies. $\sigma_1 \in \sigma^{R,W}(w)$ implies that s_1 and s_2 are optimal pure strategies.

The mixed strategy σ_2 is not optimal such that at least one of the pure strategies cannot be optimal. We know that s_1 is optimal. Therefore, s_3 cannot be optimal.

Putting these two observations together, the best pure strategies are $s^{R,W}(w) = \{s_1, s_2\}$.

Any mixture of optimal pure strategies is an optimal mixed strategy. Therefore we have $\sigma^{R,W}(w) = \left\{(\alpha, 1-\alpha, 0) \in \mathbb{R}^3_+ \,|\alpha \in [0,1]\right\}$.

Problem 3 (10 points)
Consider the quasi-linear utility function given by

$$u(x_1, x_2) = \ln x_1 + x_2 \qquad (x_1 > 0)!$$

Assume $\frac{m}{p_2} > 1$! *Hint: the household optimum is* $x(m, p) = \left(\frac{p_2}{p_1}, \frac{m}{p_2} - 1\right)$.

(a) Determine the Hicksian demand function $\chi\left(\bar{U}, p\right)$ (i.e., Hicksian demand for both goods).

(b) Determine the equivalent variation for the price increase from p_1^l to $p_1^h > p_1^l$.

Solution

a) We immediately obtain $\chi_1\left(\bar{U}, p\right) = \frac{p_2}{p_1}$ because the Marshallian demand does not depend on the income of the agent, hence the Marshallian and the Hicksian demand coincide. The Hicksian demand of player 2 $\chi_2\left(\bar{U}, p\right)$ is derived by:

$$\begin{aligned}
\bar{U} &= \ln \chi_1\left(\bar{U}, p\right) + \chi_2\left(\bar{U}, p\right) \\
&= \ln\left(\frac{p_2}{p_1}\right) + \chi_2\left(\bar{U}, p\right) \\
&\rightarrow \chi_2\left(\bar{U}, p\right) = \bar{U} - \ln\left(\frac{p_2}{p_1}\right)
\end{aligned}$$

Comment: Shepard's lemma is an alternative way to find $\chi_2\left(\bar{U}, p\right)$. You might also have used $x_2 = \frac{m}{p_2} - 1$ and $V(x(m, p)) = \ln\left(\frac{p_2}{p_1}\right) + \frac{m}{p_2} - 1$ to obtain

$$m = \left[\bar{U} + 1 - \ln\left(\frac{p_2}{p_1}\right)\right] p_2$$

and hence

$$\begin{aligned}
\chi_2\left(\bar{U}, p\right) &= \frac{\left[\bar{U} + 1 - \ln\left(\frac{p_2}{p_1}\right)\right] p_2}{p_2} - 1 \\
&= \bar{U} - \ln\left(\frac{p_2}{p_1}\right)
\end{aligned}$$

b) The equivalent variation is implicitly defined by

$$U\left(x\left(p_{new}, m\right)\right) \stackrel{!}{=} U\left(x\left(p_{old}, m+ev\right)\right).$$

Therefore,

$$\begin{aligned}
\ln\left(\frac{p_2}{p_1^h}\right) + \frac{m}{p_2} - 1 &= \ln\left(\frac{p_2}{p_1^l}\right) + \frac{m+ev}{p_2} - 1 \\
\ln p_2 - \ln p_1^h &= \ln p_2 - \ln p_1^l + \frac{ev}{p_2} \\
ev &= p_2\left(\ln\frac{p_1^l}{p_1^h}\right).
\end{aligned}$$

Comment: you might have noticed $ev < 0$.

Problem 4 (5 points)
Comment: Following a price increase of good g by one Euro, expenditure must be increased by

$$\frac{\partial e(p, \bar{u})}{\partial p_g} \leq \chi_g.$$

Solution

Denote the new price vector by $\hat{p} = (p_1, \ldots, p_g + 1, \ldots, p_\ell)$. Note $U(\chi(p, \bar{u})) = \bar{u}$. At prices $\hat{p}$, the old bundle costs $\hat{p} \cdot \chi = p \cdot \chi + 1 \cdot \chi_g$. Therefore, the expenditure increases by, at most, χ_g and we have $e(\hat{p}, \bar{u}) \leq e(p, \bar{u}) + \chi_g$ or

$$\frac{e(\hat{p}, \bar{u}) - e(p, \bar{u})}{p_g + 1 - p_g} \leq \chi_g.$$

If 'one Euro' is understood in the continuous sense (as might be suggested by the term $\partial e(p, \bar{u}) / \partial p_g$) we even know that equality holds (Shepard's lemma).

Problem 5 (7 points)
For a household with wealth A and possible loss D, the budget equation is given by

$$\frac{\gamma}{1-\gamma}x_1 + x_2 = \frac{\gamma}{1-\gamma}(A-D) + A,\ 0<\gamma<1$$

where $\gamma \cdot K$ is the payment to the insurance if K is to be paid to the insuree in case of damage D. Write the Slutsky equation for the consumption in case of damage (x_1).

Assume that x_1 is a normal good. Which conclusion can you draw for a nonnegative insurance (where the insurance increases the payoff in case of damage).

Solution

For the problem at hand, we have the Slutsky equation

$$\frac{\partial x_1^{\text{endowment}}}{\partial \frac{\gamma}{1-\gamma}} = \frac{\partial \chi_1}{\partial \frac{\gamma}{1-\gamma}} + \frac{\partial x_1^{\text{money}}}{\partial m}(A-D-\chi_1).$$

We know

- $\frac{\partial \chi_1}{\partial \frac{\gamma}{1-\gamma}} \leq 0$ (Hicksian law of demand)
- $\frac{\partial x_1^{\text{money}}}{\partial m} > 0$ (because x_1 is a normal good)
- $A-D-\chi_1 \leq 0$ (nonnegative insurance)

and therefore find

$$\frac{\partial x_1^{\text{endowment}}}{\partial \frac{\gamma}{1-\gamma}} = \underbrace{\frac{\partial \chi_1}{\partial \frac{\gamma}{1-\gamma}}}_{\leq 0} + \underbrace{\frac{\partial x_1^{\text{money}}}{\partial m}}_{>0}\underbrace{(A-D-\chi_1)}_{\leq 0},$$

and

$$\frac{\partial x_1^{\text{money}}}{\partial m}(A-D-\chi_1) \leq 0$$

which implies $\frac{\partial x_1^{\text{endowment}}}{\partial \frac{\gamma}{1-\gamma}} \leq 0$, i.e., 1 is *ordinary.*

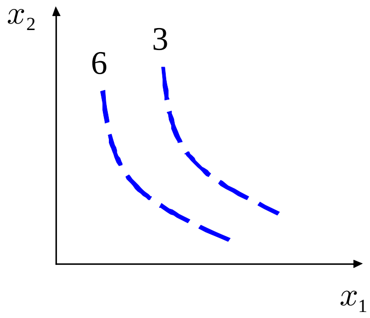


Figure 1:

Problem 6 (5 points)
Sketch indifference curves for non-monotonic and concave preferences!

Solution

The indifference curves stand for non-monotonic preferences as can be seen from $3 < 6$. Also the indifference curves hint to concavity (the worse sets are convex).

Problem 7 (8 points)
For the following decision situation,

	w_1	w_2
s_1	4	4
s_2	1	5
s_3	5	1

answer the following four questions: Are strategies s_1 and s_2 rationalizable with respect to W and/or with respect to Ω?

Solution

From

- $s_1 \notin s^{R,W}(w_1) = \{s_3\}$ because of $5 > 1$ and $5 > 4$
- $s_1 \notin s^{R,W}(w_2) = \{s_2\}$ because of $5 > 1$ and $5 > 4$

we can infer that s_1 is not rationalizable w.r.t. W.
However, s_1 is rationalizable w.r.t. Ω, $\{s_1\} = s^{R,\Omega}(1/2, 1/2)$, because

$$\begin{aligned} u(s_1, (1/2, 1/2)) &= \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 4 \\ &= 4 \\ &> 3 = u(s_2, (1/2, 1/2)) = u(s_3, (1/2, 1/2)) \end{aligned}$$

s_2 is rationalizable w.r.t. W: $s^{R,W}(w_2) = \{s_2\}$ because of $5 > 1$ and $5 > 4$.
s_2 is rationalizable w.r.t. Ω: $s^{R,W}((0, 1)) = \{s_2\}$ because of $5 > 1$ and $5 > 4$. (Note that $(0, 1) \in \Omega$ is to be identified with $s_2 \in W$!)

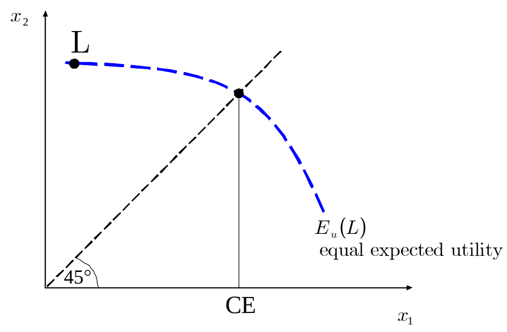


Figure 2:

Problem 8 (5 points)
Identify the certainty equivalent in the x_1-x_2 diagram for risk-loving (!) preferences.

Solution

Comment: It is important to clearly mark the indifference curve and a lottery on that indifference curve. Also, CE is a payment and not the point (CE, CE).