Microeconomics

Profit maximization

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Structure

Introduction

- Household theory
- Theory of the firm
  - Production theory
  - Cost
  - **Profit maximization**
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

Overview

- Profit maximization in input space – factor demand
- Profit maximization in output space – good supply
- Revealed profit maximization

Profit maximization in input space – factor demand

"Factor price equals marginal value product"

- Profit function:

$$\Pi(x_1, x_2) = pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

- Necessary condition for profit maximization:

$$p \cdot MP_1 \overset{!}{=} w_1$$

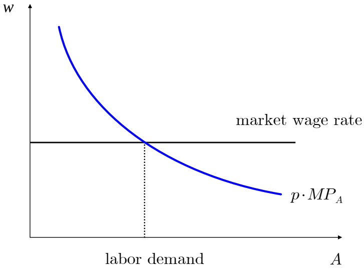
and

$$p \cdot MP_2 \overset{!}{=} w_2$$

$\Longrightarrow$ „marginal value product $\overset{!}{=}$ factor price"

Profit maximization in input space – factor demand

"Factor price equals marginal value product"



$w \uparrow \Rightarrow A \downarrow$ but

- wage = purchasing power and hence $p \uparrow$ but
  - more likely: purchasing power of products in other industries
  - profits also yield a higher demand (capital goods)
- higher wages (possibly) lead to higher productivity.

Profit maximization in input space – factor demand

Factor-demand function

- In the short run $x_2 = \overline{x_2}$:

$$\Pi(x_1, x_2) = pf(x_1, \overline{x_2}) - w_1 x_1 - w_2 \overline{x_2}$$

- Optimality condition:

$$p \cdot \frac{\partial f(x_1, \overline{x_2})}{\partial x_1} \overset{!}{=} w_1 \text{ and } p \cdot MP_1 \overset{!}{=} w_1$$

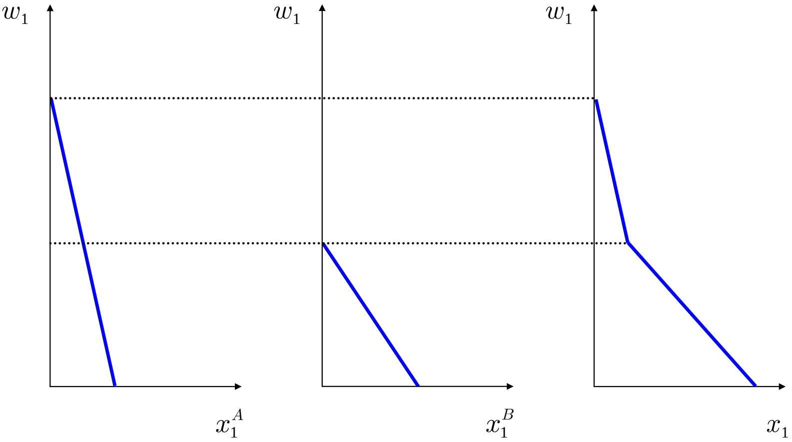
Problem

$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$

- Factor-demand function?
- Short-run factor-demand function for $\overline{x_2} = 8$?

Profit maximization in input space – factor demand

Market-demand function for factors



Profit maximization in output space – supply for goods

"Price = marginal cost"

- Profit:

$$\Pi(y) = R(y) - C(y)$$

- Optimality condition:

$$\frac{d\Pi}{dy} = \frac{dR}{dy} - \frac{dC}{dy} = MR - MC \overset{!}{=} 0$$

- Profit for fixed prices:

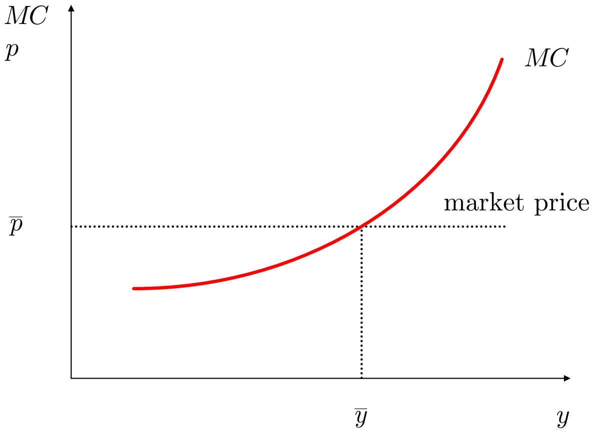
$$\Pi(y) = py - C(y)$$

- Optimality condition for fixed prices:

$$p \overset{!}{=} MC$$

Profit maximization in output space – supply for goods

"Price = marginal cost"



Profit maximization in output space – supply for goods

Supply function for goods

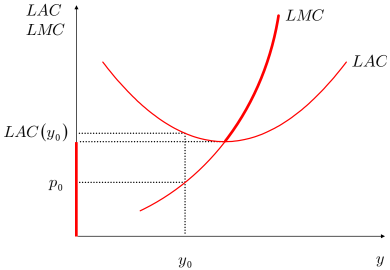
- How many units does a firm want to produce and sell for a given price:

$$y = S(p).$$

- For the most part, the supply curve equals the marginal-cost curve.
- Long-run supply: The price needs to be higher than average cost. Otherwise, output is zero.

Profit maximization in output space – supply for goods

Long-run supply for goods



for price p_0
price $=$ marginal cost
yields output y_0
and profit

$$[p_0 - LAC(y_0)]\, y_0 < 0$$

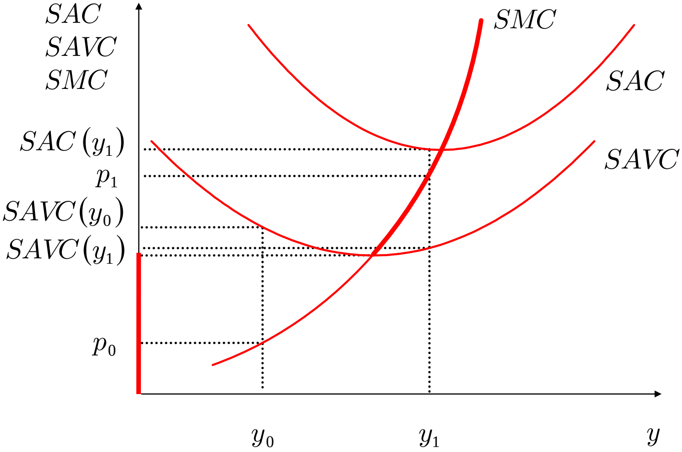
Problem

Long-run supply function for the cost function

$$C(y) = \begin{cases} 6y^2 + 15y + 54 & \text{for } y > 0 \\ 0 & \text{for } y = 0 \end{cases}$$

Profit maximization in output space – supply for goods

Short-run supply for goods



Profit maximization in output space – supply for goods

Short-run supply for goods

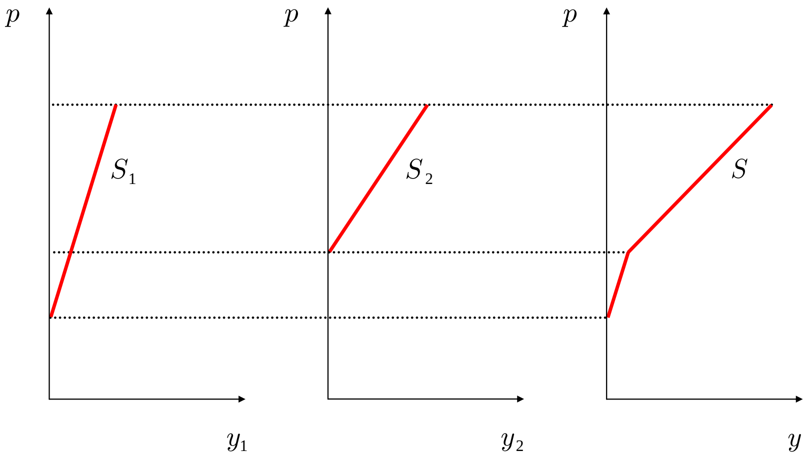
- Price $>$ average variable cost $\Longrightarrow$ firm should produce
- Price $<$ average variable cost $\Longrightarrow$ firm should not produce

Problem

A firm has the short-run cost function $C_s(y) = 6y^2 + 15y + 54$. Determine the short-run supply function!

Profit maximization in output space – supply for goods

Market-supply function



Profit maximization in output space – supply for goods

Market-supply function

Add up the supply functions $S_1, \ldots, S_n$ of all firms:

$$S(p) = S_1(p) + \cdots + S_n(p).$$

Problem

Illustrate graphically the derivation of the market-supply curve from the firms' supply curves $S_1(p) = p - 10$ and $S_2(p) = p - 15$!

Problem

The supply curve is given by $S(p) = 4p^2$. State the inverse demand curve!

Revealed profit maximization

A firm that chooses input and output at given prices reveals two things:

- The chosen input-output combination is a possible production plan.
- The chosen input-output combination yields at least the same profit as every other possible input-output combination (for the given prices). (assumption: profit maximization!)

Revealed profit maximization

Weak axiom of profit maximization

- Two periods A and B
- Output: y
- Input: x_1 and x_2
- Prices: p, w_1 and w_2.

- For prices $\left(p^A, w_1^A, w_2^A\right)$ the firm chooses $\left(y^A, x_1^A, x_2^A\right)$.
- For $\left(p^B, w_1^B, w_2^B\right)$ the firm chooses $\left(y^B, x_1^B, x_2^B\right)$.

Hence:

$$\begin{aligned} p^A y^A - w_1^A x_1^A - w_2^A x_2^A &\geq p^A y^B - w_1^A x_1^B - w_2^A x_2^B, \\ p^B y^B - w_1^B x_1^B - w_2^B x_2^B &\geq p^B y^A - w_1^B x_1^A - w_2^B x_2^A. \end{aligned}$$

Revealed profit maximization

Comparative statics

After some calculations (see text book) using

- $\Delta w_1 := w_1^A - w_1^B$
- $\Delta y := y^A - y^B$ etc.

we obtain

$$\Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0.$$

- $\Delta w_1 = \Delta w_2 = 0 \Rightarrow \Delta p \Delta y \geq 0$
  Hence, the supply curve is positively sloped
- $\Delta w_2 = \Delta p = 0 \Rightarrow -\Delta w_1 \Delta x_1 \geq 0$ or $\Delta w_1 \Delta x_1 \leq 0$
  Hence, production factors are ordinary

Central tutorial I

Problem K.5.1.

$y = f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}, w_1 = w_2 = 1$

a) Short-run supply function for $\overline{x_2} = 1$?

b) Long-run supply function?

Problem K.5.2.

Supply function $S(p) = 4p$

fixed Cost 100

Change in profit for a price increase from 10 to 20?

Problem K.5.3.

a) Short run supply curve $C_s(y) = y^2 + 1$?

b) Long run supply curve $C(y) = \begin{cases} y^2 + 1 & \text{for } y > 0 \\ 0 & \text{for } y = 0. \end{cases}$

Central tutorial II

Problem K.5.4.

Short-run cost function $C_s(y) = 300 + 3y^2$

Long-run cost function $C(y) = \begin{cases} 300 + 3y^2 & \text{for } y > 0 \\ 0 & \text{for } y = 0 \end{cases}$

a) $SAC(y)$, $SAVC(y)$, $SAFC(y)$, $SMC(y)$?

b) Minimum of short-run average cost?

c) Quantity and profit for a price of 90?

d) Quantity and profit for a price of 30?

e) Short-run and long-run supply function?

Central tutorial III

Problem K.5.5.

Farmer Lindemann owns a cow named Elsa. She gives milk according to the production function

$$y_M = f(W, G) = W^{\frac{1}{4}} G^{\frac{1}{4}},$$

where

- y_M is the milk produced,
- W is the amount of water Elsa drinks, and
- G is the amount of grass eaten by Elsa.

The prices for milk (p_M), water (p_W) and grass (p_G) cannot be influenced by Lindemann.

Determine Lindemann's demand function for water!